Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle $\tan^{-1}2$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P has travelled a distance 56 m horizontally from O, it is at a vertical height H m above the plane. When P has travelled a distance 84 m horizontally from O, it is at a vertical height $\frac{1}{2}H$ m above the plane.

Find, in either order, the value of u and the value of H. [5]

2 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A with the string taut. It is given that OA makes an angle θ with the downward vertical through O, where $\tan\theta = \frac{3}{4}$. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{5ag}$, and it starts to move along a circular path in a vertical plane. When P is at the point B, where angle AOB is a right angle, the tension in the string is T.

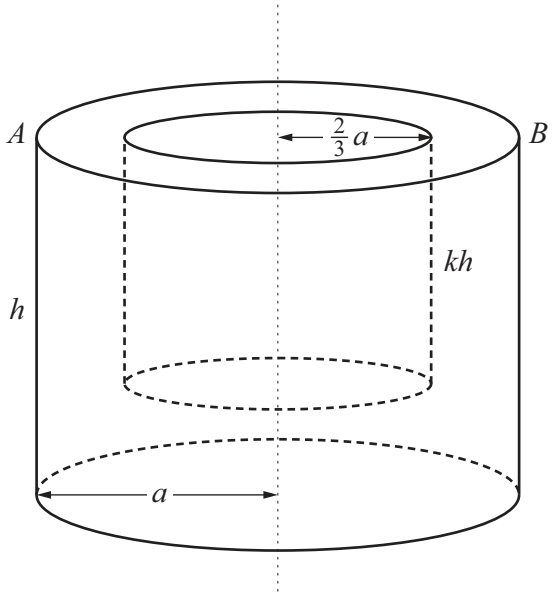
Find T in terms of m and g. [5]

3 A particle P of mass m kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity $2mg$ N. The other end of the string is attached to a fixed point O. The particle P hangs in equilibrium vertically below O. The particle P is pulled down vertically a distance d m below its equilibrium position and released from rest.

(a) Given that the particle just reaches O in the subsequent motion, find the value of d. [6]

(b) Hence find the speed of P when it is 2 m below O. [2]

4



An object is formed by removing a cylinder of radius $\frac{2}{3}a$ and height kh ($k < 1$) from a uniform solid cylinder of radius a and height h. The vertical axes of symmetry of the two cylinders coincide. The upper faces of the two cylinders are in the same plane as each other. The points A and B are the opposite ends of a diameter of the upper face of the object (see diagram).

(a) Find, in terms of h and k, the distance of the centre of mass of the object from AB. [4]

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When the object is suspended from A, the angle between AB and the vertical is θ, where $\tan\theta = \frac{3}{2}$.

(b) Given that $h = \frac{8}{3}a$, find the possible values of k. [3]

5 A particle P of mass 2 kg moving on a horizontal straight line has displacement x m from a fixed point O on the line and velocity v m s^{-1} at time t s. The only horizontal force acting on P is a variable force F N which can be expressed as a function of t. It is given that

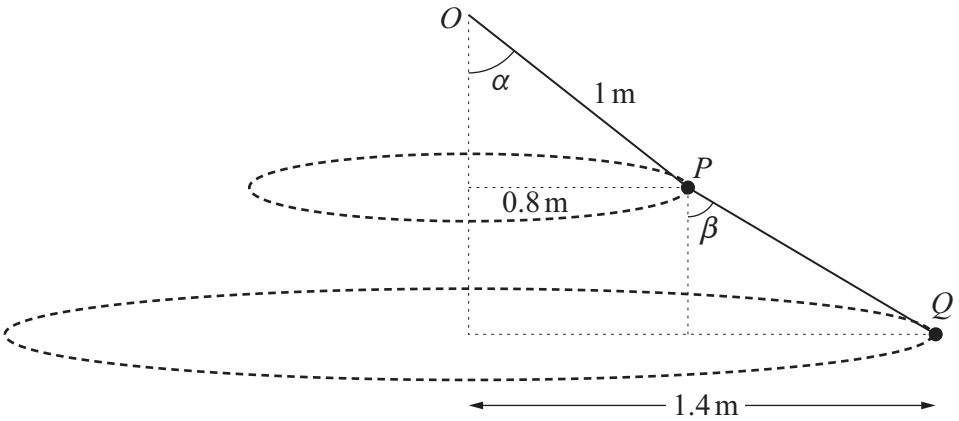
$$\frac{v}{x} = \frac{3-t}{1+t}$$

and when $t = 0$, $x = 5$.

(a) Find an expression for x in terms of t. [4]

(b) Find the magnitude of F when $t = 3$. [3]

6



A particle P of mass 0.05 kg is attached to one end of a light inextensible string of length 1 m. The other end of the string is attached to a fixed point O. A particle Q of mass 0.04 kg is attached to one end of a second light inextensible string. The other end of this string is attached to P.

The particle P moves in a horizontal circle of radius 0.8 m with angular speed ω rad s^{-1}. The particle Q moves in a horizontal circle of radius 1.4 m also with angular speed ω rad s^{-1}. The centres of the circles are vertically below O, and O, P and Q are always in the same vertical plane. The strings OP and PQ remain at constant angles α and β respectively to the vertical (see diagram).

(a) Find the tension in the string OP. [3]

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(b) Find the value of ω. [3]

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(c) Find the value of β. [2]

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7 A particle P is projected with speed u at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P is moving horizontally, it strikes a smooth inclined plane at the point A. This plane is inclined to the horizontal at an angle α, and the line of greatest slope through A lies in the vertical plane through O and A.

As a result of the impact, P moves vertically upwards. The coefficient of restitution between P and the inclined plane is e.

(a) Show that $e\tan^2\alpha = 1$. [4]

In its subsequent motion, the greatest height reached by P above A is $\frac{3}{16}$ of the vertical height of A above the horizontal plane.

(b) Find the value of e. [6]

Additional page

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